Market competitive situation: Cellular automata simulations

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Abstract

Cellular automata are mathematical models that were initially used in hard sciences, but which were then adopted by social sciences and management for the modelling of behaviour. This paper deals with the usefulness of cellular automata for the study of a competitive situation on a given market. We present the possibility of using cellular automata to carry out market studies, and for simulating the launch of new products and the withdrawal of a product. Cellular automata can allow us to understand the behaviour of agents and the implications, in terms of market share variations, of competing brands on a given market.

Keywords: Market, Cellular automata simulations, product.

Introduction

Cellular automata are mathematical models that were initially used in hard sciences, but which were then adopted by social sciences for the modelling of behaviour. They were introduced by John von Neumann and Stanislas Ulam in 1940 (under the name of "cellular spaces") for the purposes of developing simple mathematical systems that could replicate, in a manner similar to that of living organisms. In 1970, John Conway created the most well-known cellular automaton, the "Game of Life", as a representation of the survival of the species. His idea was that reproduction and the survival of the species are often governed by simple and locally effective rules, depending on each species and on the neighbouring species. Today, cellular automata are used in many disciplines, including hard sciences such as physics, chemistry and metallurgy. However, they have also found their way into social sciences. For the latter, there is a dual interest: they make it possible on the one hand to mimic the decision-making behaviour and actions of a social agent, and on the other hand to take into account the interactions between agents (cells). They are for example used in sociology to model the transmission of culture between individuals, to explain the formation of groups and conformity, or to analyse the manner in which individuals interact with each other, by exchanging their values in an effort to improve themselves, etc. (Gaylord & Nishidate, 1996). The study published by Schelling (1969) concerning spatial segregation (the advent of ghettos in the USA) for example, has been one of the most frequently commented examples (Varenne, 2011). Cellular automata (CA) are also used in management to study the global dynamics of organizational populations (Lomi and Larsen, 1996) or the interdependence of the different distribution networks in a particular sector (Liarte, 2006). By formalising simple operating rules and interaction conditions, behaviours at both micro and macro scales can be simulated. The aim of this quantitative tool is not to explain the complexity of a system, as would be the case of qualitative methods, but to simulate and observe the hypothetical consequences of these behaviours (Nowak and Lewenstein, 1996).

The objective of the present study is to explore the usefulness of CA for the study of a competitive situation on a given market. More precisely, we present the possibility of using CA to carry out market studies, and for simulating the launch of new products and the withdrawal of a product. CA can allow us to understand the behaviour of agents and the implications, in terms of market share variations, of competing brands on a given market.

The first section in this paper provides an overall presentation of CA. The second section is devoted to their use for the simulation of a market competitive situation. The third sessions shows the results of a fictitious simulation of a simplified market situation. In the discussion certain approaches are presented, which could allow this method to more accurately resemble real situations.

Cellular Automata

Each system that is analysed, in terms of a large number of discrete elements with local interactions, can be considered as a cellular automaton. CA are mathematical models in which space and time are discrete variables. They are considered to be a representation of a system with several components, each of these being in interaction with its neighbours.

What is a cellular automaton?

A cellular automaton can be defined both mathematically, and also in terms of computing science (Gaylord & Nishidate, 1996).

Mathematical definition of a CA

A CA is a discrete dynamic system in which its space, time and states are discrete and have the following characteristics:

- The space is represented by a regular grid with one, two or three dimensions (see explanations below).
- Each element (or square) of the system can adopt a finite number of states.
- The CA varies over time and the values of the elements are updated at each instant t.
- The elements are updated using certain rules, which take into account the value of each element as well as that of its neighbours.

Creating a cellular automaton is equivalent to creating a simple universe, with its own space-time structure and its own laws.

Computing science definition of a CA

A cellular automaton is a network of cells of dimension n interacting with each other, in which each cell can have k different states at each instant t (Gaylord & Nishidate, 1996, p. 23). At each instant t each cell can change state according to the CA's transition rules. The rules determine the manner in which a cell changes state at instant t, as a function of its state and that of its neighbouring cells at the instant t-1.

Construction of a cellular automaton

In order to construct a cellular automaton, it must be defined according to its three characteristics: its geometry, its neighbourhood rules, and its interaction rules.

Geometry of the cellular network:

This is the network's number of dimensions: one, two or three.

- One-dimensional CA

This geometry can be visualised as a CA having two neighbouring cells, one to the right and one to the left.

Let a_i^t be the value of the cell *i* during the period *t*.

$a_i^{t+1} = F(a_{i-1}^t, a_i^t, a_{i+1}^t)$

The state of cell *i* during the period *t*+1 will thus be a function of its own state, the state of cell *i*-1, and that of cell *i*+1, during period *t*.

- Two-dimensional CA

This geometry can be visualised as a CA having cells at all points in a plane with integral coordinates.

Let $a_{i,j}^t$ be the value of the cell *i,j* for period *t*.

$a_{i,j}^{t+1} = F(a_{i,j}^t\ a_{i-1,j}^t, a_{i,j-1}^t, a_{i+1,j}^t, a_{i,j+1}^t)$

The state of cell *i,j* at period *t*+1 will thus be a function of its own state, and of the state of cells: *i*-1,j; *i*+1,*j*; *i,j*-1; and *i,j*+1 at period *t*.

- Three-dimensional CA

This category of CA has cells at each point in a Euclidean space with integral coordinates.

The neighbourhood

The notion of neighbourhood is used here according to its meaning in computing, i.e. as all of the cells surrounding the central one, including the adjacent cells situated along the prolongation of its diagonals. In a given network, it is necessary to define how the neighbourhood of a cell can modify the cell's variations. This first requires the number of neighbours in contact with the cell to be defined.

Three types of neighbourhood have been used in the case of a two-dimensional network:

a) The von Neumann neighbourhood, in which each cell has relationships with its four respective neighbours, situated to the north, south, east and west (Fig. 1)

	N	
W -	X -	E
	S	

Figure 1: The von Neumann neighbourhood

b) The Moore neighbourhood, in which each cell has relationships with its eight respective neighbours, situated to the north, north-east, east, south-east, south, south-west, west, and north-west (Fig. 2).

NW	N	NE
W	X	E
SW	S	SE

Figure 2: The Moore neighbourhood

c) The Moore – von Neumann neighbourhood, in which each cell has relationships with four additional cells, situated to its north, east, south, and west (Fig. 3).

		Nn		
	NW	N	NE	
Ww	W	X	E	Ee
	SW	S	SE	
		Ss		

Figure 3: The Moore - von Neumann neighbourhood

In order to create a homogenous CA, the same type of neighbourhood is used for all cells.

Interaction rules between cells

The operating principle of a CA is that each cell can, at each instant, have a finite number of states, and that its state at instant *t* is a function of its interactions with the neighbouring cells. If *k* is the number of possible states for each cell, and *m* is the number of neighbouring cells, there are k^{k^m} possible interaction rules.

A CA is expressed as an algorithm. In order to construct a CA, a matrix must first be defined, in which the elements can be real numbers, natural integers, symbols or even vectors, depending on the type of system to be modelled. A function is then defined, determining the rules for variations in the value of each element (cell) of the matrix, as a function of its initial state and interaction with its neighbours.

If *n* is the dimension of the matrix, *k* the number of possible states for each cell, F the function defining cell changes, and N the values of the neighbouring cells, then the dynamics of the CA will depend on the initial state of the cells C_0, F, and N.

1. Application of cellular automata for the simulation of a market competitive situation

In order to apply CA for the simulation of a market competitive situation, one must first define the notion of space. What will the CA express, and what will the cells be made up from?

The market

According to Grover and Srinivasan (1987), we define a market as "a set of consumers with heterogeneous preferences for different brands". The heterogeneity of preferences can be explained by two factors: the heterogeneity of purchasing situations (available sum, foreseen use, place of purchase, etc.) and the structural heterogeneity in the criteria of choice (importance placed on price, or a specific characteristic, etc.).

The heterogeneity of preferences is an important element, since it is the basis of market segmentation. Market segmentation is the subdivision of this market into sub-groups comprising consumers who express identical preferences at the time of purchase. It thus results that at instant *t*, a segment is made up from buyers who have chosen the same brand.

Market dynamics then depend on the supply of each brand on the market. This supply is made up from a marketing mix (product, price, promotion, place) and exerts varying degrees of attraction on different segments. Thus, a relatively inexpensive brand of medium quality will be more attractive for an occasional buyer than for a major consumer. During each period, a certain portion of the consumers is in a purchasing context, whereas the others are not. These consumers will thus choose a brand according to their needs (type of use) and the positioning of the brands.

The market and CA

An *n*-dimensional CA thus corresponds to a market with *n* x *n* consumers. As in the case of pick/any/ models, it is assumed that the number of alternatives (different brands) evaluated by the consumers, and their identity, are known. The CA can have one or two dimensions, and the three types of neighbourhood can be used.

The cells in the CA are thus made up from "consumer – brand" pairs: a letter (A, B, C, …) defining the consumer, characterised by his/her purchasing behaviour, and a number (1, 2, 3,…) indicating the brand, characterised by its marketing mix. The consumers (A, B, C, …) are characterised by a vector U_{ijt}, which is the utility vector of brand *j* for consumer *i* and for the context of use *t*. The brands are characterised by a vector T_j which expresses the marketing mix of brand *j*. After having defined the initial states of the CA, it is still possible to modify the values of the vectors U_{ijt} and T_j. After determining the rules of the CA, including the modification of the structure of the vectors U_{ijt} and T_j after *p* iterations, we obtain:

- the variation in sales of each brand as a function of the number of iterations (purchasing periods) and the dimension *n* of the CA, or of the *n* x *n* consumers (Fig. 5).

Since the market share is indicative of the marketing competitiveness of the company (Bultez, 1996), it is possible, after each iteration, to identify the direction of change within the market share of each brand, and from this to determine what action should be taken: attack, defend, etc. (see Merunka & Roy, 1991, for other possibilities).

- Information related to what happens when a new brand enters the market or when an existing brand is withdrawn from the market (as in the case of *launching new products, the extension or disappearance of brands*).

2. Illustration

Here, we present the simulation of a competitive situation of a hypothetical market. Just as in the case of simulations carried out in social sciences and management, this makes use of a two-dimensional network, with a von Neumann neighbourhood. Moreover, this first simulation relies on the following simple rules:

- The market comprises three segments, A, B and C, in which A represents consumers who are "rational" in their choices (they compare both brands and select the best one), B represents consumers who are loyal to a certain brand (they do not change brands) and C represents the consumers who seek variety (they change brand out of the pure pleasure of changing). Although these three types of consumer are realistic (they can be observed on many different markets), they do not represent the diversity of the useable segmentations;
- Two brands, 1 and 2, with their respective mixes, with equivalent utilities;
- Each consumer chooses just one brand at a time;
- The consumers make purchases with the same frequency, by purchasing during each period.

We constructed a matrix of dimension 40 (*n*=40), i.e. 1600 consumers, with an initial population density of 2%. The population density expresses the percentage of empty cells in a CA. In our simulation of a competitive situation, this density corresponds to the number of consumers who consume neither brand 1, nor brand 2.

When the type A consumer "encounters" brands 1 and 2, he/she will select the brand with the greatest usefulness (the aim of the article is not to determine how the consumers select brands on the basis of their preferences, expectations or intention to purchase). Let us assume that brand 1 is cheaper than brand 2, and that the consumer

chooses on the basis of price only. Thus, as consumer A is very sensitive to cost, he/she will select brand 1 (Indeed, a CA allows as many variables as necessary to be used).
In a market, some segments are loyal to a certain brand, and others change their brands. The type B consumer represents a loyal consumer of brand 1 or 2. If he/she consumes brand 1, he/she will stay with brand 1, and if he/she consumes brand 2, he/she will stay with brand 2.
The type C consumer seeks variety. If the C consumer consumed brand 1, he/she will change brand, and will thus purchase brand 2, as soon as he/she discovers it, and will then come back to brand 1, at the time of the next encounter, and so forth.
Fig. 4 shows a CA (of dimension *n*=3) or a market with 9 consumers (3x3). The first diagram in this figure shows the CA's state at instant *t* (initial period). The second diagram shows the CA's state at instant *t*+1, after the first iteration. The cells A1 (type A consumer who consumes brand 1) do not change, and cells A2 (type A consumer who consumes brand 2) are transformed to A1 cells. The B1 cells (type B consumer who is loyal to brand 1) do not change. The B2 cells (type B consumer who is loyal to brand 2) do not change. The C1 cells (type C consumer who consumes brand 1) change to C2 cells (type C consumer who consumes brand 2), and *vice versa*.

A1	A1	A2
B1	C1	B1
B2	C2	C1

Situation at instant *t*+1

	A1	***A1***
B1	***C2***	B1
B2	***C1***	***C2***

(after one iteration)

Figure 4: Initial state and state after the first iteration of a CA

Results

Figs. 5 and 6 show the results after 20 iterations, simulating a competitive situation between brands 1 and 2 on a market with 1600 consumers, where each brand occupies 50% of the market at the beginning of the simulation, with a population density of 2%.

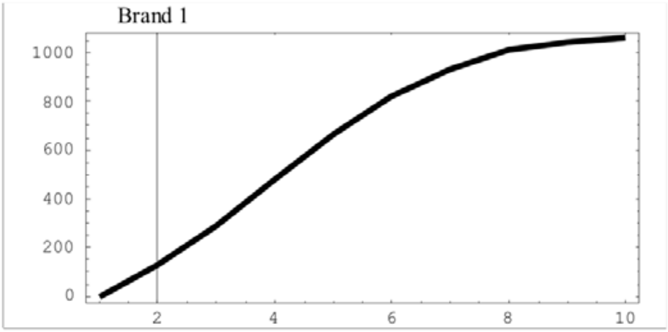

Figure 5: Variation of the sales of brand 1 when it is cheaper

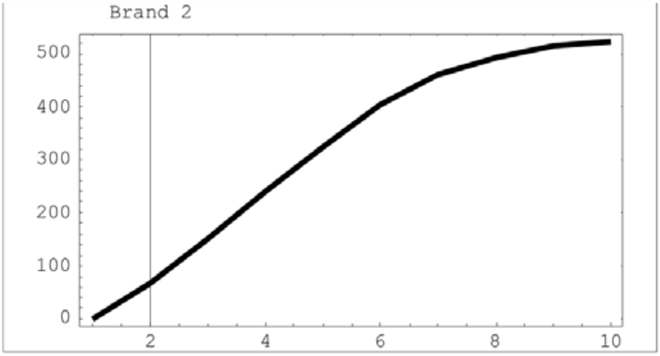

Figure 6: Variation of the sales of brand 2 when brand 1 is the cheapest

It should thus be noted that, in a context where brand 1 is cheaper than brand 2, the distribution of market shares is 72.5% for brand 1, as opposed to 37.5% for brand 2. This situation could put pressure on brand 2 to reduce its selling price. In order to illustrate the effects of such a measure, a new simulation was carried out. The conditions of this simulation are exactly the same as in the previous case, with the exception of a decrease in selling price of brand 2 after the third iteration. As a result, brand 1 remains cheaper than brand 2 when the number of iterations t is smaller than 3 ($t \leq 3$), but brand 2 becomes less expensive than brand 1 when the number of iterations is greater than 3 ($t > 3$). This change in price of brand 2 will have an influence on the behaviour of type A consumers, who – choosing brands according to their price – will tend to choose brand 1 during the first three iterations, but will then change to brand 2 after the third iteration, since this brand will then be the cheapest. The behaviours of type B (loyal) and type C (looking for variety) consumers do not change.

In order to simulate this situation, we considered the same CA with n=40 dimensions, the same population density of 2% (d=2%), but for which the rules of interaction between cells are different, depending on whether $t \leq 3$, or $t > 3$. Figures 7 and 8 show the results of this new simulation.

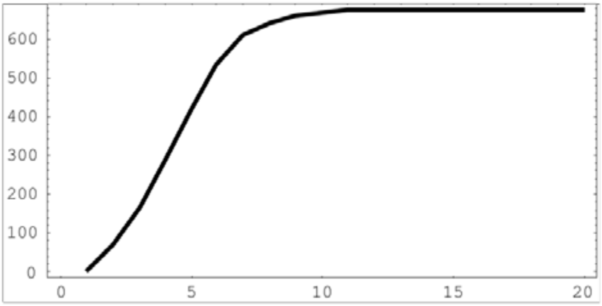

Figure 7: Variation in sales for brand 1, when it is cheaper during the first three periods, and then more expensive during the following periods.

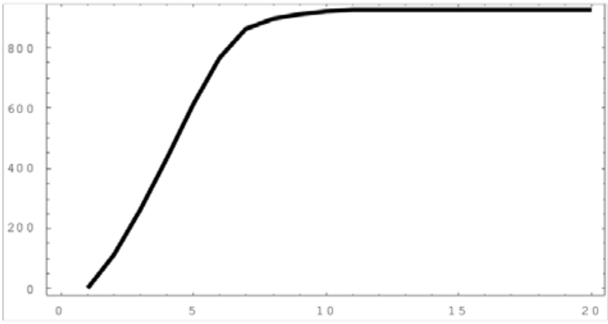

Figure 8: Variation in sales for brand 2, when this brand is cheaper during the first three periods, and then more expensive during the following periods.

These figures clearly show that the change in price of brand 2 has a significant influence on its sales volume: in the present case, its market share is 59%.

Discussion

The considerable simplicity of these simulations justifies a discussion of certain choices.

Market segmentation and CA

A very simple market structure was used in these examples: three segments and two brands. In addition, from the outset each consumer is assigned to a given segment, to which he/she remains loyal throughout the simulation. This assignment is not realistic, since a consumer can belong to several segments, mainly as a function of the situation in which he/she is purchasing.

In order to model the possibility of a consumer belonging to several segments, two possible approaches can be considered:

- The first involves assigning each consumer-cell to a purchasing situation before each iteration, taking into account the overall weighting of each purchasing situation. If for example a market is structured with respect to three purchasing situations, representing 50%, 30% and 20% of purchasing contexts, respectively, it would be possible, at the beginning of each simulation, to randomly assign 50% of buyers to the first purchasing situation, 30% to the second, and 20% to the last.
- The second approach involves dividing each CA into as many sub-CA as there

are segments. In this way, consumer heterogeneity is represented by the use of several homogeneous segments, in which each segment has a specific purchasing behaviour. The competitive structure models made the assumption that a consumer's response to a brand is independent of his/her responses for the other brands. Thus, in Dirichlet-type models – multinominal and choice map – it is assumed that each consumer has a set probability of choosing one brand for each condition, and that these choices are independent between consumers and the possible conditions (Elrod and Keane, 1995). The CA allow this limitation to be exceeded, such that they allow different conditions to be applied, in which the behaviour of a certain type of consumer in a given segment can be dependent on his/her behaviour in another segment. This means that in the CA, the probabilities of choosing a given brand do not have to be constant.

Probability of choosing brands

The choice of a brand involves a considerably more complex process than that described above. Type-A consumers do not reason just in terms of price. They are also sensitive to other criteria, such as of a brand's availability, its image and design, etc. In this context, what is the probability of a type-A consumer choosing brand 1 rather than brand 2? One could consider using a formula such as that applied by the MARKSTRAT simulation (Larréché and Gratignon, 1990), where the probability of choosing a brand is expressed as follows

$$\text{Probability of purchasing brand 1 in segment S} = \frac{\text{Effective distribution coverage of brand 1 in segment S} \; \text{X} \; \text{Index of purchasing intention for brand 1 in segment S}}{\text{Effective distribution coverage of all brands in segment S} \; \text{X} \; \text{Index of purchasing intention for all brands in segment S}}$$

:

Time

The computer provides us with a result in a few seconds. However, the question to be answered is how to translate these seconds into real time, and how to express the variation of sales in terms of real time. To solve this problem we considered that one iteration could express the time interval between two purchases. That implies that this time will depend on the type of product, and that it will vary for different products, and thus for different CAs.

Purchasing frequency

Although we have assumed that all consumers purchase a given brand at a constant rate, it would be more realistic to consider that for each period; only a certain fraction of consumers really makes a purchase.

Population density

An important element for CA simulations is the initial population density, which expresses the percentage of empty cells in the CA. In our simulation of the competitive situation of a market, this density is the number of consumers who consume neither brand 1, nor brand 2. This means whether the market is saturated or not. The following figures show how the sales of brand 1 change as a function of the initial population density.

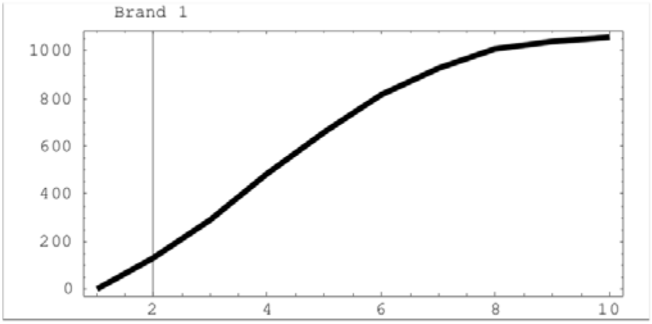

Figure 9: Example of the variation in sales of brand 1, in which this brand is less expensive than brand 2, with an initial population density of 2%.

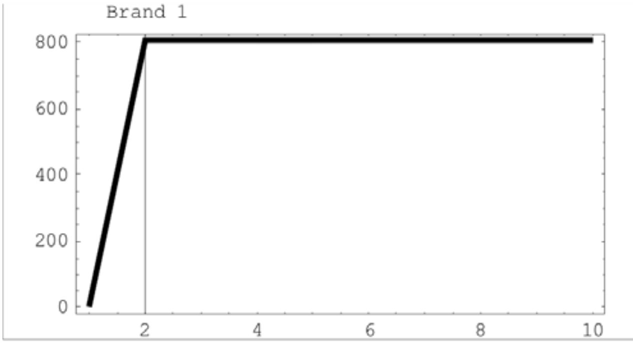

Figure 10: Example of the variation in sales of brand 1, in which this brand is less expensive than brand 2, with an initial population density of 80%.

Conclusions

CA are simulation tools for various real phenomena. They can provide astonishing results in certain fields. Some CA with similar transition functions (rules) can thus become very different, whereas other CA with different functions can become identical. We plan to simulate different market conditions using CA, and to observe the consequences in competitive functioning. We are of the opinion that CA can allow market structures, the variation of a brand's market share, and more importantly the launching of a new brand or the disappearance of an obsolete brand, to be simulated. It is important, for the validity of the simulation, to make use of real market data, provided by *ad hoc* studies or consumer panels. It would in effect be more realistic to initialise the model with real data such as the number of brands, the structure of the marketing mix, the number of segments, the size of each segment, and its consumption style. After having stipulated the initial market conditions, it is then necessary to provide the most realistic details for the transition rules. This is certainly the most delicate phase of this development.

It is thus necessary to provide more a detailed description of the complex rules and numerous constraints of the market, in order to come as close as possible to an accurate representation of any real situation. This should nevertheless be achieved without losing sight of the fact that reality is always more complex than even the most sophisticated model.

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