How can we use a mathematical model in order to analyze and improve the efficiency of a productive activity?

Mariana Nikolla
Agricultural University of Tirana, Albania

Elfrida Dishmema
Agricultural University of Tirana, Albania

Orkida Totojani
Agricultural University of Tirana, Albania

Bedri Dragusha
University of Pristina, Kosovo

Abstract

Beer business industry is one of the most profitable agro-processing industries in the country. It has increased by expanding local and international market. There are 80 small and large beer factories that have their activity in Albania. The leading domestic beer manufacturers are: "Stela" beer, "Tirana" beer, "Korca" beer, "Kaon" beer, "Norga" beer, etc. They have invested millions of euros in terms of improving the quality of their production. Stela beer was the first private beer active in the Albanian market. Its factory production founders of Stela beer responded to the market by investing in the development of technology, modern equipment and science. Stela beer processing and production systems have been improved continuously. The investments today are at around 20 million dollars. The factory has a considerable number of employees and a production capacity of around 250,000 hl per year. This article uses information obtained for beer production during the period 2003-2016. Through non-parametric mathematical model Dea, the impact of production factors such as investment, advertising, expenses, capital and number of employees in the production of beer during the study period is analyzed. The analysis showed that the best years or more efficient years in the use of the quantity of inputs for the period 2003-2016 are 2007, 2014 and 2015. The years 2003, 2010 and 2012 are less efficient by Dea analysis. The study also showed the best possible combinations of inputs improving the efficiency of inefficient years in the Stela Beer production.

Keywords: mathematical model, production, efficiency, Stela beer, Albania.

Introduction

One of the strategic policies of the Ministry of Agriculture, Rural Development and Water Management of the Republic of Albania is to promote the development of the agro-food sector.

Increasing the quality of agricultural products, livestock and agroprocessing in the country increases the interest in these products in foreign markets. Statistics published by the Ministry of Agriculture, Rural Development and Water Management for 2016

show that Albania is seeking and finding resources to increase the production of products for export. According to official data for the first 6 months of 2016, exports of agricultural products, livestock and food processing have increased by 28% compared with a year ago. Despite the difficulties encountered in recent years, the agri-food sector has a gradual and sustainable development. One of the successful businesses of the agro-food sector, which has been increasing steadily expanding in local and international market, is the business of the beer industry.

Albanians are good consumers of domestic beer. In Albania, the beer industry began to flourish in the 30s, with the production of Korca and Tirana beer, which were called by the name corresponding to the cities where the beer was produced.

Large investments are made continuously in terms of improving its production technology. The demand for this drink is high, so the market often sees new brands of beer and businessmen who want to invest in this sector. According to the Ministry of Agriculture, in 2014, 17 companies operate in the production of beer, compared to 15 in 2013. Today beer industry is experiencing a great development due to increased production and export to the international market. Exports of beer for the first 3 months of 2016 increased by 4.5 times compared to the same period of 2015. Concretely, from 51.4 tons that were exported in the period January-March 2015, in 2016 more than 231.8 tons of beer was exported. Beer exports in 2016 registered the amount of 24 million leks, compared to 3.1 million leks in 2015. Exports are mainly concentrated in regional countries.[1] Today in Albania operate 80 factories. Largest producers of beer are Tirana beer, Stela beer, Norga beer, Kaon and Korca beer. In terms of the most popular beer brands imported in the Albanian market are: Haineken, Amstel, Loven, Brown, Foster and Peroni. Greek beers occupy main place in Albania's imports. About 42% of imports of beer in 2014 were dominated by Greece with a slight increase compared to 2013, when Greece accounted for 41%. In recent years the domestic beer is competed by the Kosovo "Peja" beer. Despite the increased import of beer; investments in local production have increased. According to the producers of beer in 2015, investment in this area in the whole place has a value of 80 million euros, while have been employed about 3 thousand persons in manufacturing and distribution [Agroweb-Albania]. The introduction of new beers on the market, according to local manufacturers is made in terms of unfair competition. For example, Kosovo "Peja" beer pay the same excise as beer produced in the country, does not change the excise tax on amounts over 200 hectoliters, thus providing cheaper price due to favorable taxation. Beer production companies in the country demand by the Albanian government to intervene for the implementation of taxation on the basis of the import or production. The total value of the sector's contribution in the state budget through taxes in 2014 amounted to 15 million euro, while in 2015 proved to be even higher, due to increased taxes.[2] But despite the high taxes, the beer business continues to develop further. One of the most successful producers over the year's local beer is "Stela" beer, produced by the company stakeholders, Stefani & CO. After 90 years, the founders of the beer factory production responded to the market hunger by investing in equipment, technology and science. Stela beer processing

[1] Albanian Ministry of Finance.
[2] Albanian Ministry of Finance.

and production systems are modern. Albanian market is plenty of beer production capacities and has been invested millions of dollars. Year after year the company expanded being tested as one of the best products in Europe. Stela beer is the first private beer in the Albanian market and accounts for about 20% of the domestic beer market. In addition, are produced soft drinks, energy drinks, mix drink, such as "Whiskey Cola", "Tequila", "Rum Cola", "Quench", "All Sports" preferred by athletes' dhe people who deal with tourism, etc. To meet different customer requirements beer producers have created a diversified portfolio. Stela Beer is rich in vitamins B, rich in antioxidants, high alcohols and easily assimilated. One of the company's products or beer is Stela Krudo. This product is a 100% natural beer with fresh yeast. Another product is Stela Dark: porter, easy preferred especially by women. Czech Pilseneris a typical Czech beer. It is characterized by a mild bitterness and lupola aroma. Stela Pils is lager type beer (light) fresh, brilliant and consistent foam. Strong Beer is a clear beer with bread and malt alcoholic. Beer is clear, transparent, colored; caramel has dense foam and balanced bitterness. The key success of Stefani & Co throughout its history is the investment in terms of increased quality, continuous improvement of production equipment and procedures, improving the image and fulfillment of customer requirements. What distinguishes Stela beer from other brands of beer produced in the country is maintaining its authenticity. The target associated with the production of this beer is the trinomial: quality, taste and health. In its function is the selection of raw materials, processing and quality control that is subject to management systems ISO and HACCP. Production technology unlike other brands of beer produced in the country has to be one of the most popular beers and worthy of consumer confidence.

Materials and Methods

The information enabled the realization of this study was obtained from various sources, such as:

- Data published by the Ministry of Agriculture, Rural Development and Water Administration in Albania;
- Surveys conducted by specialists from breweries in Albania;
- Data collected by the company stakeholders, Stefani & CO;
- Publications and studies conducted about the beer business in Albania.

Information collected for the production of Stela beer during 2003-2016 and the costs incurred for investment, advertising, capital and number of employees was subjected to statistical analysis. To analyze and improve production efficiency of this activity was used a mathematical model, non parametric Dea, a methodology based on linear programming. The model used to achieve the purpose of the study is oriented to inputs, to minimize the production cost. Model requires the assignment of output or outputs and inputs which will be used in the model. Concretely, selected output is the production of beer and inputs are: investments, advertising expenses, the number of

$$h_0(u,v) = \frac{weighted\ sum\ of\ unit\ output\ U_0}{weighted\ sum\ of\ inputs\ unit\ U_0} = \frac{\sum_{r=1}^{s} y_{r0} \times u_r}{\sum_{i=1}^{m} x_{i0} \times v_i}$$

lea, is given by

The importance of using this model lies in assignment of weights (portions) u_r u_r and v_i v_i which represent the decision variables in the problem solving linear [illegible] set the objective function for unit $U_0, U_0,$ which is the same $Max: \sum_{r=1}^{s} y_{r0} \times u_r$dy:

with appropriate restrictions for each unit that investigated:

1. $\sum_{r=1}^{s} y_{r\_} u_r - \sum_{i=1}^{m} x_{i\_} v_i \leq 0 \sum_{r=1}^{s} y_{r_0} u_r - \sum_{i=1}^{m} x_{i_0} v_i \leq 0$
2. $\sum_{i=1}^{m} x_{i_0} v_i = 1 \sum_{i=1}^{m} x_{i_0} v_i = 1$
3. $u_r \geq 0$ *and* $v_i \geq 0 ,\forall r\, u_r \geq 0$ *and* $v_i \geq 0 ,\forall r$ and *i. i.*

Implementation of the model is done in Excel, Data and Solver Solution. Units that are efficient will have outcome 100% Dea efficiency. Efficient units are the best choice in the group.
If coefficient $h_0 h_0$ is less than 1, the unit is relatively inefficient. The model is further used to determine the best weights of the inputs used to obtain at least the same output. [1],[2]

Implementation of the model

The following model has been applied to assess the efficiency of the unit 1 (2003).
PL problem for the study unit is as follows:
Max: 97864 $u_1 u_1$ output per unit weight of 1.
Conditions:

$$\begin{cases} 97864u_1 - 112v_1 - 831244v_2 - 10510v_3 - 9957v_4 \leq 0 & condition\ of\ efficiency\ of\ unit\ 1 \\ 110231u_1 - 110v_1 - 832187v_2 - 10210v_3 - 10234\ v_4 \leq 0 & condition\ of\ efficiency\ of\ unit\ 2 \\ 109734u_1 - 110v_1 - 829650v_2 - 9620v_3 - 9957\ v_4 \leq 0 & condition\ of efficiency\ of\ unit\ 3 \\ \ldots\ldots & \\ 100000u_1 - 80v_1 - 874000v_2 - 6967v_3 - 0v_4 \leq 0 & condition\ of efficiency\ of\ unit\ 16 \\ 112v_1 + 831244v_2 + 10510v_3 + 9957\ v_4 = 1 & condition\ of efficiency\ of\ unit\ 1 \\ u_1, v_1, v_2, v_3, v_4 \geq 0 & conditions\ not\ negativity \end{cases}$$

Problem solving is done by a computer program Excel (Data, Solver Solution). [3], [4], [5]

Findings and their interpretation

In Figure 1 are presented the results of the Dea analysis of efficiency, unit 8 (2010) and other units. By applying the Solver, we find the optimal solution for unit 8 (2010) which results in efficient Dea equal to 75%. To complete the analysis of efficiency for other units is needed to change the content of the cell "Unit" (C21) manually 1, 2, 3, 4, …, 14 and optimize new worksheet Solver for each unit and align their efficiencies in the "Dea efficiency".
The efficiency analysis in this way is not very comfortable when analyzing a very large number of units. In such cases, create a macro in Excel button, the execution of

which implements the process automatically, with the click of a button.

Figure 1. The results of the Dea analysis for the study units.

C21 fx 8

	A	B	C	D	E	F	G	H	I	J	K
1	Years	Units	Output1	Input1	Input2	Input3	Input4	Weight	Weight	Diference	Efficiency
2	Y.	U.	Production	Nr.of employeers	Capital	Expenses for advertising	Investments	Outputs	Inputes	≤0	Dea%
3	2003	1	97864	112	831244	10510	9957	0.712	1.116	-0.404	71%
4	2004	2	110231	110	832187	10210	10234	0.801	1.084	-0.282	80%
5	2005	3	109734	110	829650	9620	9523	0.798	1.021	-0.223	81%
6	2006	4	121439	106	825980	11689	8980	0.883	1.241	-0.358	90%
7	2007	5	136314	110	826840	9963	10150	0.991	1.058	-0.067	100%
8	2008	6	107063	112	827360	10780	9870	0.778	1.144	-0.366	79%
9	2009	7	107289	112	835600	11052	11560	0.780	1.173	-0.393	78%
10	2010	8	103428	101	863500	9420	10789	0.752	1.000	-0.248	75%
11	2011	9	107070	101	860000	9820	10876	0.778	1.042	-0.264	77%
12	2012	10	102880	101	859200	9350	10889	0.748	0.993	-0.245	75%
13	2013	11	112625	90	865000	9760	11078	0.819	1.036	-0.217	81%
14	2014	12	136812	85	852000	10025	0	0.995	1.064	-0.070	100%
15	2015	13	138127	85	865000	9460	0	1.004	1.004	0.000	100%
16	2016	14	100000	80	874000	6967	0	0.727	0.740	-0.013	98%
17											
18		Weigh	0.000	0.000	0.000	0.000	0.000				
19							Variable Cells				
20							Max: C22 Variables: C18:G18 Constrains: C23=1 C18:G18 >=0 J3:J16<=0				
21		Unit	8		Set Cell						
22		Outpu	0.75		Constraint Cells						
23		Input	1.00								

Source: Data provided by the company Stefani & Co and processed by the authors.

Further, the model provides the opportunity to improve efficiency for inefficient years, showing the best combination of inputs, to receive at least the same production in lower cost. The analysis resulted in units 1 (year 2003), 8 (year 2010) and 10 (year 2012) are less efficient than other units of the group, respectively $h_0(u,v) = 71\%$ $h_0(u,v) = 71\%$ and 75%.

To improve their efficiency acted as follows:

At first, solve the Dea problem for the units above; select the option Sensitivity Report in the Solver Results dialog box. In the results of the Sensitivity Report, the absolute value of shadow prices in the column "Difference", are weights of composed unit. The hypothetical unit produces output equivalent to the output of the unit under study seeking smaller amounts or equivalent to those used by inefficient units. In Figure 2 are presented the results of sensitivity analyse per unit 8 (year 2010), while in Figure 3 is presented the best combination of inputs that make unit 8 efficient by Dea. The improved Dea efficiency for unit 8 is showed in figure 4.

Figure 2. The results of the sensitivity of the unit 8 (year 2010)

Microsoft Excel 14.0 Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C18	Weights Prodhimi	7.27E-06	0	103428	1E+30	103428
D18	Weights Nr.punonjesve	0	-12.301763	0	12.30176328	1E+30
E18	Weights Kapitali	0	-1622.3765	0	1622.376509	1E+30
F18	Weights Shpenz.Reklama	0.0001062	0	0	1E+30	17.69865282
G18	Weights Investimet	0	-8112.9906	0	8112.990624	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C23	Input Prodhimi	1.000	**0.752**	1	1E+30	1.000
J3	≤0	-0.404	**0.000**	0	1E+30	0.404
J4	≤0	-0.282	**0.000**	0	1E+30	0.282
J5	≤0	-0.223	**0.000**	0	1E+30	0.223
J6	≤0	-0.358	**0.000**	0	1E+30	0.358
J7	≤0	-0.067	**0.000**	0	1E+30	0.067
J8	≤0	-0.366	**0.000**	0	1E+30	0.366
J9	≤0	-0.393	**0.000**	0	1E+30	0.393
J10	≤0	-0.248	**0.000**	0	1E+30	0.248
J11	≤0	-0.264	**0.000**	0	1E+30	0.264
J12	≤0	-0.245	**0.000**	0	1E+30	0.245
J13	≤0	-0.217	**0.000**	0	1E+30	0.217
J14	≤0	-0.070	**0.000**	0	1E+30	0.070
J15	≤0	0.000	**0.749**	0	0.017336315	1.004
J16	≤0	-0.013	**0.000**	0	1E+30	0.013

Source: Data processed by the authors.

Figure 3. The Composite unit values for unit 8 (year 2010)

Years	Units	Output1	Input1	Input2	Input3	Input4	Weight	Weight	Diference	Efficiency
Y.	U.	Production	Nr.of employeers	Capital	Expenses for advertising	Investments	Output	Inputes	≤0	Dea%
2003	1	97864	112	831244	10510	9957	0.946	1.329	-0.383	**71%**
2004	2	110231	110	832187	10210	10234	1.066	1.328	-0.263	**80%**
2005	3	109734	110	829650	9620	9523	1.061	1.316	-0.255	**81%**
2006	4	121439	106	825980	11689	8980	1.174	1.331	-0.157	**90%**
2007	5	136314	110	826840	9963	10150	1.318	1.318	0.000	**100%**
2008	6	107063	112	827360	10780	9870	1.035	1.326	-0.291	**79%**
2009	7	107289	112	835600	11052	11560	1.037	1.346	-0.309	**78%**
2010	**8**	**103428**	**64**	**647703**	**7084**	**0**	**1.000**	**1.000**	**0.000**	**100%**
2011	9	107070	101	860000	9820	10876	1.035	1.366	-0.331	**77%**
2012	10	102880	101	859200	9350	10889	0.995	1.360	-0.365	**75%**
2013	11	112625	90	865000	9760	11078	1.089	1.373	-0.284	**81%**
2014	12	136812	85	852000	10025	0	1.323	1.323	0.000	**100%**
2015	13	138127	85	865000	9460	0	1.335	1.335	0.000	**100%**
2016	14	100000	80	874000	6967	0	0.967	1.322	-0.356	**98%**
Weight		0.000	0.000	0.000	0.000	0.000				
Unit		8								
Output		1.00								
Input		1.00								

Variable Cells

Max: C22
Variables: C18:G18
Constrains: C23=1
C18:G18 >=0
J3:J16<=0

Solver Constraint Cells

Source: Data proc thors

re 4. Improved D for unit 8.

Source: Data processed by the authors

Years	Units	Output1	Input1	Input2	Input3	Input4	Weight	Weight
Y.	U.	Production	Nr.of employeers	Capital	Expenses for advertising	Investments	Value	%
2003	1	97864	112	831244	10510	9957	**0.00**	**0%**
2004	2	110231	110	832187	10210	10234	**0.00**	**0%**
2005	3	109734	110	829650	9620	9523	**0.00**	**0%**
2006	4	121439.4	106	825980	11689	8980	**0.00**	**0%**
2007	5	81788.4	110	826840	9963	10150	**0.00**	**0%**
2008	6	107062.6	112	827360	10780	9870	**0.00**	**0%**
2009	7	107289.1	112	835600	11052	11560	**0.00**	**0%**
2010	**8**	**103428**	**101**	**863500**	**9420**	**10789**	**0.00**	**0%**
2011	9	107070	101	860000	9820	10876	**0.00**	**0%**
2012	10	102880	101	859200	9350	10889	**0.00**	**0%**
2013	11	112625	90	865000	9760	11078	**0.00**	**0%**
2014	12	136812	85	852000	10025	0	**0.00**	**0%**
2015	13	138127	85	865000	9460	0	**0.75**	**75%**
2016	14	100000	80	874000	6967	0	**0.00**	**0%**
The composition values								
Extra inputs		**103428**	**64**	**647703**	**7084**	**0**		
used		**0**	**37**	**215797**	**2336**	**10789**		

Conclusions and Recommendations

The use of mathematical models in business direction helps managers to go inside economic phenomena. They are important for the development of economic processing, programs and boundary determination in the increasing production data quantity of production factors. These production factors are determined by optimal levels of production, so that the resultant index get maximum value, such as production, productivity, etc., or the minimum value as cost, etc. In this paper was used a mathematical model, non parametric Dea to analyze the efficiency of production of one of the required products in the domestic market but not only, as is "Stela" beer. Solving the problem of linear programming through Dea model, which is displayed in Figure 1, show those units: 5, 12 and 13 have Dea 100% efficiency, while other units in the study are inefficient. For inefficient units, Dea gives the possibility of finding the best alternative using the inputs to produce at least the same output. For example, unit 8 represented by year 2010, has a coefficient 75% of efficiency, so it is inefficient by Dea analysis. Referring to the report of the sensitivity results, the absolute value of shadow prices are composed of the unit weights, which resulted more efficient than in the study of unit 8. This unit produces the same output 103428 hl seeking smaller amounts of inputs (labor, capital, expenditures for advertising), used by the company in 2010 (figure 4). Such similar analysis can also be used for manufacturing activities in various branches of economy.

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